## A Computational Perspective on the Typology of Agreement

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## Variation in Agreement

Focusing on $\phi$-agreement...

- Which elements agree?
- Probes: T/C/v
- Goals: All DPs/some DPs
- What elements can intervene?
- Minimality effects
- Misc. blockers, e.g. finite C
- What are the positions of source and target?
- Probe c-commands goal
- Goal c-commands probe


## Some Puzzles for Agree

- Why should visibility vary?
- Why should there be blockers, and why should they vary? (cf. Halpert 2019; Keine 2020)
- Why should directionality vary? Does it really? (cf. Pesetsky and Torrego 2007; Zeijlstra 2012)
- Why does a probe sometimes agree with multiple goals? (cf. Deal 2015, et seq.)


## Overview of the Talk

Most long-distance linguistic dependencies are in the formal class tier-based strictly local (TSL) (Heinz 2018; Graf 2022a).

- Long-distance phonotactics (Heinz 2018)
- Movement (Graf 2022b)
- Case licensing (Vu et al. 2019; Hanson 2023)

Claim: Syntactic agreement is also TSL.
Why this matters:

- Limits predicted structural configurations
- Provides parameters for variation


## Computational Intuitions

Strictly local (SL): constraints on sequences of adjacent elements

- Phonology: local phonotactics
- No consonant clusters! (*CC)
- No vowel hiatus! (*VV)
- No voiceless consonant after a nasal! (*NT)
- Syntax: selection, functional hierarchies
- Selection: object of devour must be a DP!
- Functional hierarchy: $\mathrm{T}<$ (Perf) $<$ (Prog) $<$ (Pass) $<\mathrm{V}$


## Computational Intuitions (2)

Tier-based strictly local (TSL): constraints on sequences of adjacent elements... when the irrelevant elements are ignored

- Phonology: vowel harmony (ignore intervening consonants)
- ex. front-back harmony
$\checkmark$ kubulo $x$ kibilo
*[+back][-back], *[-back][+back]
- Syntax: subject-verb agreement (ignore things other than finite T and D)
- ex. There seem to be some ducks in the garden.
* $T_{S G} D_{P L}$ * $T_{P L} D_{S G}$


## Limits on Structural Configurations

TSL patterns can relate elements at a distance, but are otherwise severely restricted in what they can do.

- No arbitrary logic - "you can have A...B...C or X...YY...Z, but not both"
- No counting - "you can have A...B...C, but only up to three times"


## Parameters for Variation

The space of possible TSL constraints corresponds neatly to variation in long-distance dependencies.

- Visibility: which elements are relevant and which are ignored?
- Blocking: are there elements which block dependency formation?
- Directionality: do we ban XY, YX, or both?


# Parameters for Variation (2) 

| Phenomenon | $\phi$-agreement | Vowel harmony |
| :--- | :--- | :--- |
| Participants | Probe and most DPs | Most vowels |
| Invisible | Non-DPs, some DPs | Consonants, some vowels |
| Blockers | Finite C, some DPs | Some vowels |
| Directionality | Downward/upward | Progressive/regressive |

## What Else Can TSL Do?

- Selective opacity
- probe horizons (Keine 2020)
- One probe sharing multiple goals
- e.g. interaction/satisfaction theory (Deal 2015)
- Two elements interacting within some domain
- e.g. dependent case (Baker 2015)
- Conjoined vs independent probes (cf. Lohninger et al. 2022)


## Roadmap

- SL and TSL formal languages
- Constraints on syntactic derivations
- Formal typology of agreement
- Invisibility
- Blocking
- Multiple probes
- Directionality
- Multiple goals


## SL and TSL Formal Languages

## Strictly Local Languages

In a strictly $k$-local (SL-k) language, a string is well-formed iff it does not contain any forbidden substrings of some fixed length $k$.

- $\Sigma=$ "alphabet" = set of all symbols
- $G=$ "grammar" = forbidden substrings


## Example: CV alternation (SL-2)

$\Sigma=\{\mathrm{C}, \mathrm{V}\} \quad G=\{\mathrm{VV}, \mathrm{CC}\}$
Licit words: CVC, VCV, CVCVC, ...
Illicit words: CVVC, CVCCV, CVVCCV ...

## Strictly Local Languages (2)

To model constraints at the start/end of a word, we add edge markers $x / \propto$ and use them in the grammar like any other symbol.

Example: CV syllables, optional final C (SL-2)
$\Sigma=\{\mathrm{C}, \mathrm{V}\} \quad G=\{\times \mathrm{V}, \mathrm{VV}, \mathrm{CC}\}$

Illicit words: $\not \mathrm{VCV}_{\ltimes}, \ngtr \mathrm{CVV} \ltimes, \nsim \operatorname{CVCCV} \ltimes, \ldots$

## Tier-Based Strictly Local Languages

In a tier-based strictly $k$-local (TSL-k) language, a string is well-formed iff its tier projection does not contain any forbidden substrings of some length $k$.

- $T=$ "tier alphabet" = set of salient/visible symbols


## Example: Vowel harmony (TSL-2)

Front-back harmony, 'e' is transparent, 'a' is a blocker

$$
\begin{aligned}
& \Sigma=\{\mathrm{k}, \mathrm{~b}, \mathrm{l}, \mathrm{i}, \mathrm{u}, \mathrm{o}\} \\
& T=\{\mathrm{i}, \mathrm{u}, \mathrm{o}\} \\
& G=\{\mathrm{iu}, \mathrm{io}, \mathrm{oi}, \mathrm{ui}\}
\end{aligned}
$$

$$
\begin{aligned}
& \rtimes \text { kubulo } \\
& \rtimes k i b i l o \ltimes \\
& \begin{array}{l}
\checkmark \\
x
\end{array}
\end{aligned}
$$

## Tier－Based Strictly Local Languages（2）

## A more complex example

Front－back harmony，＇e＇is transparent，＇a＇is a blocker

$$
\begin{aligned}
& \Sigma=\{\mathrm{k}, \mathrm{~b}, \mathrm{l}, \mathrm{i}, \mathrm{e}, \mathrm{u}, \mathrm{o}, \mathrm{a}\} \\
& T=\{\mathrm{i}, \mathrm{u}, \mathrm{o}, \mathrm{a}\} \\
& G=\{\mathrm{iu}, \mathrm{io}, \mathrm{oi}, \mathrm{ui}\}
\end{aligned}
$$

| Word | Tier |
| :---: | :---: |
| kubulo | 入uu0x |
| kibilo | xiiox |
| kubelo | 入uox |
| kibelo | メiox |
| ubalo | »uaox |
| kibalo | «iao |

Constraints on Syntactic Derivations

## EPP Movement

(1) Minimality
a. The cat $\left[{ }_{v} \mathrm{P} \quad\right.$ chases the rats].
b. *The rats ${ }_{v p} \underset{x}{ }$ the cat chase $\left.\underset{\square}{ }\right]$.
(2) Blocking
a. This student seems [TP $\quad$ to be a genius].
b. * This student seems $\underset{x}{\text { [cp that }} \__{\square}$ is a genius.]

## Derivation Trees

'The cat chases the rats.'


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## Derivation Trees

'The cat chases the rats.'


## Derivation Trees (2)

The rightmost child of a node is its complement; others are specifiers.



See Graf and Kostyszyn (2021) for details. Related: Brody (2000).

## Minimality

$\checkmark$ The cat [ ${ }_{v} \mathrm{P}$ _ chases the rats]. vs. $X$ The rats $\left[{ }_{v} \mathrm{P}\right.$ the cat chase __].


## Blocking

$\checkmark$ This student seems [Tp __ to be a genius].
$\boldsymbol{x}$ This student seems [cp that ___ is a genius.]


## TSL Grammar for EPP Movement

Constraints:

- Every EPP landing site should immediately followed by an EPP mover on the tier, and vice versa.
- No potential EPP-related element may intervene.
- No blocking elements may intervene.

TSL grammar:

- Project a tier with all nodes of categories T/D/C
- Banned substrings:

$$
\left\{\begin{array}{ll}
X_{[+E P P]} \cdot X_{[+E P P]} & X_{[-E P P]} \cdot X_{[-E P P]} \\
X_{[+E P P]} \cdot X & X \cdot X_{[-E P P]} \\
X_{[+E P P]} \cdot \ltimes & \rtimes \cdot X_{[-E P P]}
\end{array}\right\}
$$

## Subject-Verb Agreement

(3) Minimality
a. The cat chases the rats. (subject agreement)
b. * The cat chase the rats. (object agreement)
(4) Long-distance agreement
a. Some ducks seem to be in the garden.
b. There seem to be some ducks in the garden.
(5) Finite C blocks agreement
a. It seems that there are some ducks in the garden.
b. * It seem that there are some ducks in the garden.
(6) Finite $C$ is not always opaque
a. Nobody said that there are any ducks in the garden.
b. * Somebody said that there are any ducks in the garden.

## Agreement and Minimality

$\checkmark$ The cat chases the rats. (subject agreement)
$x$ The cat chase the rats. (object agreement)


## TSL Grammar for Subject-Verb Agreement

Constraints:

- Every $\phi$-probe site should immediately followed by a $\phi$-goal on the tier, and vice versa.
- No potential $\phi$-related element may intervene.
- No blocking elements may intervene.

TSL grammar:

- Project a tier with all nodes of categories T/D/C
- Banned substrings:

$$
\left\{\begin{array}{ll}
X_{[+\phi]} \cdot X_{[+\phi]} & X_{[-\phi]} \cdot X_{[-\phi]} \\
X_{[+\phi]} \cdot X & X \cdot X_{[-\phi]} \\
X_{[+\phi]} \cdot \ltimes & \rtimes \cdot X_{[-\phi]}
\end{array}\right\}
$$

## Command Strings

A command string (c-string) is a derivational ordering of nodes.

- There is a c-string from the root to each node.
- Among each head and its arguments: Head < Specifier < Complement.


See Graf and Shafiei (2019) for details. Related: Frank and Vijay-Shankar (2001).

## Command Strings (2)

We're interested in c-strings that trace the complement spine of the tree, or of a left branch.


See Graf and De Santo (2019) regarding how to distinguish spines.

## Tiers Over Command Strings

$\checkmark$ The cat chases the rats. (subject agreement)


## Tiers Over Command Strings (2)

$x$ The cat chase the rats. (object agreement)


The Typology of Agreement

## Parameters for Variation

TSL patterns have two types of parameters:

- Which elements are projected on the tier?
- What are the local constraints on the tier?

| $\left.\begin{array}{ll}\text { Participants } & \text { Probe and most DPs } \\ \begin{array}{l}\text { Invisible } \\ \text { Blockers }\end{array} & \begin{array}{l}\text { Non-DPs, some DPs } \\ \text { Some DPs, finite C }\end{array} \\ \hline \begin{array}{l}\text { Directionality } \\ \text { Multiple agreement }\end{array} & \begin{array}{l}\text { Downward/upward } \\ \text { One/multiple } \\ \text { probes/goals }\end{array}\end{array}\right\}$ Tier projection |
| :--- | :--- |

## Case Studies

1. Invisibility: Case-sensitive agreement (Hindi)
2. Blocking: Dative intervention (Icelandic)
3. Multiple Probes: Complementizer agreement (West Flemish)
4. Directionality: More complementizer agreement (Lubukusu)
5. Multiple goals: Existential clauses (English)

Invisibility

## Case-Sensitive Agreement

In Hindi, the verb agrees with the closest nominative argument, which may not be the subject.
(7) Hindi verbal agreement ignores ergatives (Mahajan 1990)
a. Raam rotii khaataa thaa.

Raam.м.nom bread.f.nom eat.IPFV.m be.pst.m
'Raam ate bread (habitually).'
b. Raam-ne roTii khaayii.

Raam.M-ERG bread.F.NOM eat.PFV.F
'Raam ate bread.'
Analysis: Project D only if nominative. Tier constraints are unchanged.

## Case-Sensitive Agreement (2)

'Raam ate bread (habitually).' (Nominative subject, subject agrees)


## Case-Sensitive Agreement (3)

'Raam ate bread.' (Ergative subject, object agrees)


## Case-Sensitive Agreement (4)

Analysis: Project D only if nominative. Tier constraints are unchanged.

| Subject Case | T agrees w/ | $\checkmark$ | Tier |
| :---: | :---: | :---: | :---: |
| Nominative | Subject | $\checkmark$ | $\mathrm{T}_{[+\phi]} \cdot \mathrm{D}_{[\text {nом },-\phi]} \cdot \mathrm{D}_{[\text {noм }]}$ |
|  | Object | $x$ | $\mathrm{T}_{[+\phi]} \cdot \mathrm{D}_{[\text {nom }]} \cdot \mathrm{D}_{[\text {Nom },-\phi]}$ |
| Ergative | Subject | $x$ | $\mathrm{T}_{\left[^{+} \phi\right]} \cdot \mathrm{D}_{[\text {nom }]}$ |
|  | Object | $\checkmark$ | $\mathrm{T}_{[+\phi]} \cdot \mathrm{D}_{[\text {nom },-\phi]}$ |

## Ergative $=$ Invisible

Oblique case-marked DPs are not necessarily invisible.
(8) Case-insensitive agreement in Nepali (Coon and Parker 2019)
a. Maile yas pasal-mā patrikā̄ kin-ē.

1sG.ERG DEM store-LOC newspaper.ABS buy-1sG
'I bought the newspaper in this store.'
b. Ma thag- $\overline{\mathrm{i}}-\overline{\mathrm{e}}$.

1sG.ABS cheat-PASS-1sG
'I was cheated.'
Analysis: Exactly as in English.

## Blocking

## Blocking and Defaults

In principle, there are two possible outcomes when agreement is blocked (cf. Preminger 2014):

1. The derivation crashes.
2. Default agreement occurs.

We will look at a case of default agreement.

## Dative Intervention

Often, datives are invisible (like ergatives in Hindi). In Icelandic, they are usually invisible, but not always.
(9) Optional agreement across dative subject (Holmberg and Hróarsdóttir 2003) a. Einhverjum stúdent finnst [tölvurnar ljótar]. some student.sG.DAT find.sG computer.PL.DEF.NOM ugly.NOM
b. Einhverjum stúdent finnast [tölvurnar ljótar]. some student.SG.DAT find.PL computer.PL.DEF.NOM ugly.NOM 'Some student finds the computers ugly.'

## Dative Intervention (2)

(10) Icelandic transitive expletive construction (Holmberg and Hróarsdóttir 2003)
a. Pað finnst einhverjum stúdent [tölvurnar ljótar]. EXPL find.DFLT some student.DAT computer.PL.DEF.NOM ugly.NOM
b. * Pað finnast einhverjum stúdent [tölvurnar ljótar]. EXPL find.PL some student.DAT computer.PL.DEF.NOM ugly.NOM 'Some student finds the computers ugly.'

Analysis of blocking data: Dative DPs do project. Probe can be followed by a non-agreeing dative. (The full pattern also TSL.)

## Dative Intervention (3)

$x$ 'There find.PL some student the computers ugly.'


## Dative Intervention (4)

$\checkmark$ 'There find.DFLT some student the computers ugly.'


## Dative Intervention (5)

Analysis: Dative DPs do project. Probe can be immediately followed by a non-agreeing dative.

- Tier projection is as in English.
- Don't ban all $X_{[+\phi]} \cdot X$, only $X_{[+\phi]} \cdot X_{[n o m]}$.

Alternative: Default agreement is agreement with the dative DP.

Multiple Probes

## Multiple Probes

- So far we've only dealt with a single $\phi$-probe in a clause.
- In general, each probe gets its own tier with its own constraints.
- It is possible, and sometimes necessary, for two probes to share a tier.


## Complementizer Agreement

In some languages with agreeing complementizers, both C and T agree with the same DP.
(11) Complementizer Agreement in West Flemish (Diercks 2013)
a. Kpeinzen da-j (gie) morgen goat.

I-think that-you (you) tomorrow go
'I think that you'll go tomorrow.'
b. Kvinden dan [die boeken]te diere zyn.

I-find that-pL[the books] too expensive are
'I find those books too expensive.'
Single-tier analysis: Relax the constraint against sequential probes.

## Complementizer Agreement (2)

'I find that the books are too expensive.'


## Complementizer Agreement (3)

Analysis: Relax the constraint against sequential probes.

- Tier projection: as in English.
- Constraints: as in English, but don't ban $X_{[+\phi]} \cdot X_{[+\phi]}$
- Or at least, don't ban $\mathrm{C}_{[+\phi]} \cdot \mathrm{T}_{[+\phi]}$

Alternative: Each type of $\phi$-probe (C, T, etc.) gets its own tier.

Directionality

## Upward Complementizer Agreement

(12) Complementizer Agreement in Lubukusu (Diercks 2013)
a. Ba-ba-ndu ba-bolela Alfredi ba-li a-kha-khile. C2-C2-people C2-said C1.Alfred C2-that C1-FUT-conquer 'The people told Alfred that he will win.'
b. Alfredi ka-bolela ba-ba-ndu a-li ba-kha-khile. C1.Alfred C1-said C2-C2-people C1-that C2-FUT-conquer 'Alfred told the people that they will win.'

## Analysis:

- Allow $\phi$-probe on C follow its goal.
- Agreement on C is subject oriented, so project only DPs with -EPP.


## Upward Complementizer Agreement (2)

'The people told Alfred that he will win.'


## Upward Complementizer Agreement (3)

Analysis: Allow $\phi$-probe on C follow its goal. Project DPs only if [-EPP].

- Project: all T, D if [-EPP], all C
- Constraints: as in English, but allow $\mathrm{D}_{[-\phi]} \cdot \mathrm{C}_{[+\phi]}$

Multiple Goals

## Multiple Goals

- Sometimes a single elements seems to get its features from several different goals, e.g. omnivorous agreement (cf. Nevins 2011).
- The interaction-satisfaction theory (Deal 2015) modifies the AGREE algorithm as follows:
- We distinguish two sets of features, the interaction set and the satisfaction set.
- A probe copies features from elements in the interaction set, but only stops once it finds an element in the satisfaction set.
- The morphology can realize the features of any/all of the elements the probe has acquired.
- The theory has many other uses, including some cases of optionality.


## Optionality

(13) Optional agreement in English existential clauses
a. There seem(s) to be some squirrels in the attic.
b. Some squirrels seem(*s) to be in the attic.

## Analysis:

- Singular/default agreement is agreement with there, whose $\phi$-features are in the interaction set but not the satisfaction set.
- Allow sequence of goals between the probe and the goal that 'satisfies' it.


## Optionality (2)

'There seem(s) to be some squirrels in the attic.'


## Optionality (3)

Analysis: Allow sequence of goals between the probe and the goal that 'satisfies' it.

- Tier alphabet: as usual
- Constraints: as usual, but allow there ${ }_{[-\phi]} \cdot D_{[-\phi]}$

Note: We could also use this analysis for dative intervention.

## Summary

| Phenomenon | Example | Tier Projection | Tier Constraints |
| :--- | :--- | :--- | :--- |
| Minimality | Subject-verb <br> agreement | All T/D/C | Strict matching of $+\phi$ <br> and $-\phi$ |
| Invisibility | Case-sensitive agree- <br> ment | All T/C <br> D only if right case | - |
|  | Subject-oriented <br> agreement | All T/C <br> D only if -EPP | - |
| Blocking | Dative intervention | - | Non-agreeing dative <br> may follow $+\phi$ |
| Multiple probes | Agreeing T \& C | - | Allow sequential $+\phi$ |
| Directionality | Upward agreement | - | Swap order of $+\phi /-\phi$ |
| Multiple goals | Optionality | - | Allow sequential $-\phi$ |

## Summary (2)

- Agreement patterns in syntax are TSL over c-strings.
- If we vary the tier projection and constraints slightly, we can account for variation across languages and constructions.
- The range of variation is similar to other phenomena, especially phonologyical harmony.
- Most of the logical possibilities of TSL are realized just within $\phi$-agreement - this is not necessarily expected!


## Open Questions

- Any TSL-3 patterns? TSL-4?
- To what extent are multiple tiers required? (subfeatures of $\phi$, subject+object agreement)
- Are there patterns that are not TSL under any reasonable analysis?
- To what extent do other kinds of agreement (e.g. negative concord) look like $\phi$ agreement?
- To the extent that movement/case/agreement are not alike, why?
- How far can we take the parallel with harmony in phonology?


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Extras

## Three Models of Locality

Finitely bounded


Unbounded


Finitely bounded with invisibility


## Computing SL/TSL Patterns

Some ways to determine whether a string satisfies a SL/TSL grammar:

1. Collect the set of length- $k$ (tier) substrings, and intersect it with the grammar. The string is well-formed iff this intersection is the empty set.
2. Read one symbol at a time, keeping track of the most recent $k-1$ (tier) symbols. Check for violations at each step. The string is well-formed if we reach the end with no violations.

## Computational Complexity

(T)SL languages are efficient to process.

- The size of the grammar is at most $|\Sigma|^{k}$, where $\Sigma$ is the set of symbols.
- Testing or generating a string takes linear time, e.g. when implemented as a finite state machine.


## Computational Complexity (2)

(T)SL languages are easy to learn.

- Just keep track of all attested (tier) substrings of size $k$.
$\rightarrow$ string extension learning (Garcia et al. 1990; Heinz 2010)
- The time to process the input data is linear.
- Very little data is needed (compared to more expressive classes).


## The Chomsky Hierarchy

Syntax is "mildly context sensitive" when analyzed over surface strings. It becomes subregular when analyzed over derivation trees.


## The Subregular Hierarchy



